Math 10B with Professor Stankova
Worksheet, Discussion #11; Thursday, 2/22/2018
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1 Random Variables

1.1 Concepts

1. A **random variable** is any function $X : \Omega \to \mathbb{R}$. It isolates some concept that we care about. For example, when we flip a coin 20 times, then we can define a random variable which is the number of heads that we flip.

   A **probability mass function (PMF)** is a function from $\mathbb{R}$ to $[0, 1]$ that is associated to a random variable X. We define $f(x) = P(X = x) = P(X^{-1}(\{x\}))$.

   Two random variables X, Y are called **independent** if for any subsets $E, F \subset \mathbb{R}$, the subsets $X^{-1}(E), Y^{-1}(F) \subset \Omega$ are independent. To prove that two random variables are independent, we need to show that those two sets are independent for any two choices of E, F (actually, it suffices to only consider E, F as one point sets or that $P(X = x, Y = y) = P(X = x)P(Y = y)$ for any $x, y \in \mathbb{R}$). To prove that they are not independent, we only need to find one counterexample pair E, F.

1.2 Examples

2. Suppose that we roll two die and let X be equal to the maximum of the two rolls. Find $P(X \in \{1, 3, 5\})$ and draw the PMF for X.

3. When rolling two die, let Y be equal to the first die roll. Are X, Y independent random variables?

1.3 Problems

4. True False A RV goes from subsets of Ω to $\mathbb{R}$.

5. True False Similar to the probability function, a PMF takes events or subsets of $\mathbb{R}$ and assigns a probability between $[0, 1]$.

6. I flip a fair coin 4 times. Let X be the number of heads I get. Draw the PMF for X.

7. I roll two fair four sided die with sides numbered $1 - 4$. Let X be the product of the two numbers rolled. Find the range of X and draw the PMF for X.

8. (Challenge) I draw 5 cards from a deck of cards. Let X be the number of hearts I draw. What is the range of X and draw the PMF of X. Use this to find the probability that I draw at least 2 hearts.